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Plasmas in extreme electromagnetic fields

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Abstract The recent developments in the generation of laser pulses with ultra-high power (presently petawatt and progressing) have opened up a new frontier in plasma research by making it possible to obtain and to study "mesoscopic" amounts of relativistic (ionized) matter in compact-size experiments in the laboratory. This will make it possible to investigate in a controlled environment the nonlinear dynamics of collective relativistic systems, to enter the Quantum Electrodynamics plasma regime, and to explore conditions that are of interest for high energy astrophysics and beyond.

Keywords Relativistic plasmas · Ultraintense electromagnetic fields · High energy laboratory astrophysics

1 Introduction

The Nobel Prize in Physics 2018 was awarded [1] "for groundbreaking inventions in the field of laser physics" with one half to Arthur Ashkin"for the optical tweezers and their application to biological systems", the other half jointly to Gérard Mourou and Donna Strickland "for their method of generating high-intensity, ultra-short optical pulses."

Strickland and Mourou's invention, the so-called "chirped pulse amplification" in the optical range [2], has driven an unprecedented development in the study of the interaction of ultra-intense electromagnetic radiation with matter [3]. In the short time period of two decades this has raced through the nonlinear optics of bound electrons in strong fields, the relativistic optics where the quiver

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velocity of the free electrons nears the speed of light, reaching the ultrarelativistic regime where the kinetic energy of the electrons is very much larger than their rest mass energy and massive ions enter the relativistic domain, and finally now aiming at the yet unexplored plasma regime where Quantum Electrodynamics effects and particle pair creation play a major role [4]. This race has opened up a new frontier in plasma research (see the "Extreme Light Infrastructure" project [5]) directing it towards the study of novel fundamental physics phenomena, such as the nonlinear dynamics of collective relativistic systems made up of high energy particles and photons that interact according to the rules of multi-particle Quantum Electrodynamics [6], and making it possible to investigate matter conditions [7] that are expected to occur in extremely energetic astrophysical objects such as pulsar atmospheres or black hole neighbourhoods. In addition it has made possible the realization of novel compact ultra-intense sources of high frequency electromagnetic radiation up to the gamma range (see e.g. [8]) and the development of high field gradient particle accelerators [9, 10] that bring promise both for high energy particle physics [11] and for medical applications such as widely distributed centres for hadron therapy [12].

2 High energy plasmas

Plasmas take a variety of forms, from magnetic loops [13] in the solar corona to giant relativistic jets [14] in galaxies, forms that are shaped by the collective dynamics of fields and particles [15]. In high temperature diluted (fully ionized) plasmas this dynamics occurs on timescales that can be very much shorter than those of the binary processes among individual particles. This dynamics is dominated by nonlinearities that under proper conditions can produce extremely large energy densities (see e.g., [16, 17]), on mesoscopic scales in the laboratory (spatial size of the order of tens of microns, particle density $10^{21} - 10^{22}\ cm^{-3}$ close to solid density, electron energy in the GeV range, ion energy presently in the tens of MeV range) and on huge spatial scales in the Universe.

2.1 High energy density

A plasma is a medium that is very effective both for producing and for manipulating [18] high energy density electromagnetic fields and electromagnetic radiation. Relativistic electromagnetic solitons [19, 20] generated in laboratory plasmas by ultraintense laser pulses provide a good example. In these plasma structures the radiation pressure of the oscillatory electromagnetic fields trapped inside the soliton acts on the plasma electrons piled up in the soliton walls. Here the radiation pressure is balanced by the Maxwell stress tension produced by the stationary electric field due to the charge separation that is created inside the soliton by the displacement of the electrons with respect to the much heavier ions. A similar phenomenon is the so-called relativistic self-focusing [21] now routinely observed [22] in ultra-intense laser plasma interaction experiments. Relativistic self-focusing allows a laser pulse to propagate without spreading over distances significantly longer than the "Rayleigh length" because light diffraction is counteracted by nonlinear effects (see [23]) and in particular by the nonlinearity introduced by the relativistic kinematics into the plasma dynamics through the relationship between particle velocities and particle momenta. This nonlinearity makes the electrons in the plasma effectively "heavier" where the laser pulse is stronger, thus causing a reduction of the local value of the effective Langmuir frequency.

2.2 Transport

The fact that plasmas can store large energy densities lies at the heart of magnetic fusion research [24] that aims to produce net thermonuclear energy in a controlled way in the laboratory at relatively low particle densities. However, the effectiveness of the collective interactions between the particles and the electromagnetic fields in plasmas can cause a fast energy transfer between particles and fields: in almost all conditions the transport properties of a fusion plasma are determined by collective plasma excitations [25], not by binary processes. Being far from thermodynamic equilibrium conditions, the amplitude of these particle and field fluctuations is orders of magnitude larger than that of the thermal fluctuations one encounters in regimes of local equilibrium, giving rise to anomalous, i.e. enhanced, transport [26]. We may just briefly recall here that the physics of thermonuclear fusion in the laboratory is very different from that inside a star because of its miniaturization [27]. A star is dense and thus opaque to the electromagnetic radiation: it is at local thermodynamic equilibrium and loses energy mostly through Black Body radiation. These losses are proportional to the star surface while gains (thermonuclear fusion power) are proportional to the star volume. To be hot and confined, a plasma in a laboratory magnetic fusion experiment must be dilute: it is transparent to its own radiation (mainly Bremsstrahlung) and suffers intrinsic losses that are proportional to the plasma volume as are its fusion gains.

The transport properties of astrophysical plasmas too can be controlled by collective plasma excitations, not by binary processes. Accretion disks [28] are an example. Conservation of angular momentum would prevent accretion of matter rotating in a disk around, e.g., a compact stellar object. Collisional (so-called molecular) viscosity is insufficient to explain the rate of accretion required in order to account for the emitted electromagnetic radiation: angular momentum must be transported outwards much more efficiently [29,30]. Collective plasma excitations, such as the so-called Magneto-Rotational (Velikhov) instability [31,32], provide the mechanism that may account for the anomalous transport of angular momentum.

Collective instabilities can also provide an effective mechanism of particle acceleration under both laboratory and astrophysical conditions. For example, a relativistic reformulation of the reconnection instability of magnetic field lines, which in nonrelativistic regimes is a major player in determining the magnetic structure of laboratory [33] and solar plasmas [34], has been invoked [35] as the acceleration mechanism of electrons up to PeV energies in the gamma-ray flares [36] observed in the Crab Nebula.

2.3 The Weibel instability

However, in determining transport properties collective excitations work differently from collisions. The Weibel instability [37] and its related quasistatic magnetic field generation provide a good example. This well known instability feeds on the energy difference between the different degrees of freedom in momentum space, just as a thermal machine would do by extracting heat from bodies at different temperatures. The major difference is in the “final” state. Collisions would equalize the temperature of the two bodies. In contrast, the Weibel instability transforms part (corresponding approximately to energy equipartition) of the energy difference between the different degrees of freedom into (steady) magnetic field energy (in a sense, this corresponds to the work that is produced by the thermal machine). Moreover a real steady state is not reached after the instability amplitude has ceased growing and the plasma continues to evolve transferring e.g., energy to smaller scale excitations as it does in a turbulent cascade process [38].

The Weibel instability is an important mechanism of magnetic field generation in space in conditions where collisional effects such as the so-called Biermann battery [39] are ineffective. It may provide the seed field that feeds the dynamo process that can produce magnetic fields on large spatial scales [40]. A variant of the Weibel instability is of major importance for laser-produced relativistic plasmas and it occurs in plasmas where an effective anisotropy in momentum space can be associated to the presence of two counterstreaming particle populations. In this case it goes under the more appropriate name of “current filamentation” instability [41,42]. Counterstreaming electron populations occur naturally in the interaction of ultraintense laser pulses with the plasma that is formed almost instantaneously when such pulses interact with a gas or with solid material: electrons are accelerated to relativistic energies by the pulse and penetrate the material and the resulting charge imbalance induces a neutralizing return current. In particular, this filamentation is of primary importance for electron transport in inertial fusion experiments and for the so-called fast ignition [43] where the generated magnetic field can channel the electrons. In very different conditions, having been reformulated within a Quantum-ChromoDynamic framework [44], it has been reconsidered under the name of “Chromo-Weibel instability” in relativistic heavy ion collision experiments.

2.4 Relativistic mirrors

Possibly the most impressive example of how relativistic plasmas can be used to concentrate electromagnetic energy is provided by relativistic plasma mirrors [45] moving at a speed close to that of light, finally performing in the laboratory the "gedanken" experiment described by A. Einstein more than a century ago [46]. The coherent interaction between a high intensity electromagnetic pulse and a foil of relativistic electrons counterpropagating at a relativistic velocity [47] provides a method for reaching unprecedented electromagnetic energy densities. The foil of counterpropagating relativistic electrons consists of the electrons accelerated at wave breaking by the electric field of a nonlinear Langmuir wake wave generated in the plasma by a second laser pulse propagating in the opposite direction with respect to the first [45]. In a longitudinal nonlinear wave (such as the large amplitude Langmuir wake wave with phase velocity close to the speed of light c induced in the plasma by the second laser pulse), if electrons are accelerated along the direction of the wave propagation up to velocities larger than the wave phase velocity, the wave breaks [48]. At the wave break position a cusp develops in the electron density and the electron oscillatory motion in the wake wave is transformed into a net relativistic motion in the direction of breaking. There are important issues that need to be addressed in the use of such relativistic mirrors, such as the reflectivity of the mirror formed by the electron foil [49], its parabolic shape that can focus the reflected pulse, etc.. However the main physics point to be noted is that the reflected pulse is not the result of the incoherent superposition of (inverse) Thomson (Compton) scattered waves but of the coherent response of the plasma electrons that, as in a Free Electron Laser, satisfy the density-wavelength condition $n\lambda^3 >> 1$. In these conditions electromagnetic processes are described by the coherent plasma response and not by single particle scattering cross sections.

Plasma relativistic mirrors provide an innovative tool for reaching higher and higher electromagnetic field intensities: this tool is based on the plasma's ability to concentrate the electromagnetic energy of pulses produced by available laser systems [47] without the need to resort to larger and larger laser systems. An example of this capability is offered by the interaction of regular nonlinear structures in the plasma, such as solitons or magnetic vortices, with a strong wake wave [50]. The electromagnetic fields of the nonlinear structures are partially reflected by the electron density modulations of the wake wave incident with a phase velocity close to the speed of light and a single-cycle, high-intensity electromagnetic pulse is formed in the reflection process. Due to the Doppler effect, the size of the reflected pulse is much shorter (it can reach the attosecond range) than that of the initial nonlinear structure [50]. As noted in [51], "the measurement and control of subcycle field evolution of few-cycle light have opened the door to a radically new approach to exploring and controlling processes of the microcosm."

3 A new plasma physics regime

These theoretical and experimental developments will make it possible to investigate a new regime of plasma dynamics where photons form an additional particle population and particle pairs can be created together with incoherent, high frequency radiation [52]. Using, as mentioned above, plasma mirrors to intensify light it will be possible to achieve extremely high electromagnetic fields [47] with amplitudes that approach the critical field of Quantum Electrodynamics, the so-called Schwinger field [53], and to create globally neutral electron-positron-photon plasmas [54], turning the vacuum into a medium that can affect the propagation of the laser pulses themselves [55].

The nonlinear dynamics of the electromagnetic field in vacuum is described in the long wavelength limit by the Euler-Heisenberg Lagrangian [56] that depends on the electromagnetic Lorentz invariants $|\mathbf{E}|^2 - |\mathbf{B}|^2$ and $\mathbf{E} \cdot \mathbf{B}$, which vanish for a plane wave in vacuum. Strongly focused and multiple intersecting pulse configurations are thus of special interest (see e.g. [57,58]) and an extensive research effort has flourished in recent years. Vacuum birefringence [59], high harmonic generation [60,61], pair production avalanches [62] and photon splitting [63] are among the phenomena that have been intensively investigated. In a plasma it will be possible to study under controlled conditions the collective dynamics of matter made of relativistic electrons and positrons [64], of interest for high energy astrophysics, in the presence of extremely large electromagnetic fields and more generally the collective dynamics of matter in a Quantum Electrodynamics regime.

4 Conclusions

The availability in the laboratory of relativistic plasmas that can be studied in compact experiments has opened up the way to rapid new developments in a wide range of diverse fields of science. To name a few, these range from nonlinear system dynamics and nonlinear relativistic optics to novel powerful sources of high energy radiation, X-ray lasers, attosecond pulses, high field gradient techniques for particle acceleration and plasmas dominated by high frequency incoherent radiation. These developments will establish a strong connection with high energy astrophysics in fields such as radiation pressure acceleration, relativistic shocks and relativistic magnetic reconnection in the search for mechanisms of extreme particle acceleration (e.g., in the context of cosmic ray physics). Using plasmas as tools, it will be possible to generate electromagnetic fields approaching the Schwinger field to investigate nonlinear optics in vacuum in the laboratory.

The miniaturization of high energy physics experiments down to “table top” size in high energy density plasmas produced by ultra-intense laser pulses has represented a major breakthrough in plasma physics research, making fast progress possible in yet unexplored regimes with a vast participation of scientists working in research institutions in different countries.

In this paper an effort has been made to provide an overall view of a vast set of topics and their intertwining relationships, together with an adequate list of references to guide the interested reader. Of course, the selection of the papers quoted reflects the personal view of the author and is not meant to be exhaustive.

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